CHAPTER 7

CURVED BEAMS

7.0. INTRODUCTION

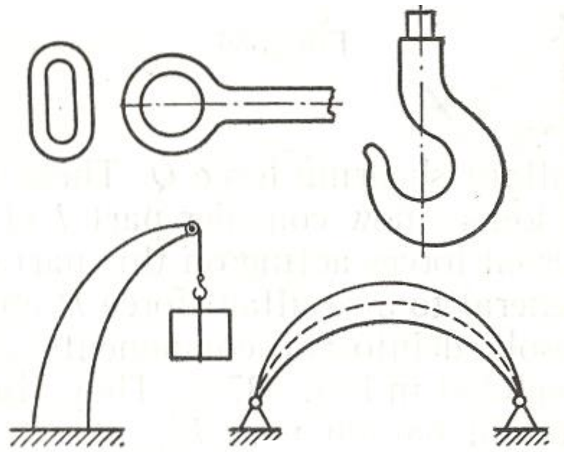


Fig. 7.1

Besides straight-axis beams in structures we often come across beams in which the axis, i.e. the line passing through the centers of gravity of successive cross sections, is a curved line. Chain links, Jugs, hooks, arches, vaults, hoisting crane frames, etc., all belong to this group of elements (Fig. 7.1). Strictly speaking, no beam has an absolutely straight axis. All beams, which we design as straight beams, have a slight curvature. Therefore, a study of the effect of curvature of the beam's axis on the distribution of stresses will, on the one hand, enable us to check the strength of beams having appreciable curvature and, on the other hand, judge the influence of a slight deviation of the axis from a straight line on the strength of a straight beam.

A curved beam is defined, as a beam in which the neutral axis in unloaded condition is curved instead of straight. Following assumptions are made in the stress analysis of curved beam:

i) Plane sections perpendicular to the axis of beam remain plane after bending.

ii) The moduli of elasticity in tension and compression are equal.

iii) The material is homogeneous and obeys Hooke's law.

7.1. STRESSES IN CURVED BEAM

The distribution of stresses in a curved beam is shown in Fig. 7.1. There are two factors, which distinguish the analysis of straight and curved beams. They are as follows:

(i) The neutral and centroidal axis of the straight beam are coincident. However, in Curved beam the neutral axis is shifted towards the center of curvature.

(ii) The bending stresses, in straight beam, vary linearly with the distance from the neutral axis. However, in curved beams, the stress distribution is hyperbolic.

Following notations are used in Fig. 7.2:

R_o= radius of outer fibre (mm)

R_i= radius of inner fibre (mm)

R = radius of centroidal axis (mm)

R_N = radius of neutral axis (mm)

h_i= distance of inner fibre from neutral axis (mm)

h_o = distance of outer fibre from neutral axis (mm)

M_b = bending moment with respect to centroidal axis (N-mm)

A = area of the cross-section (mm^2)

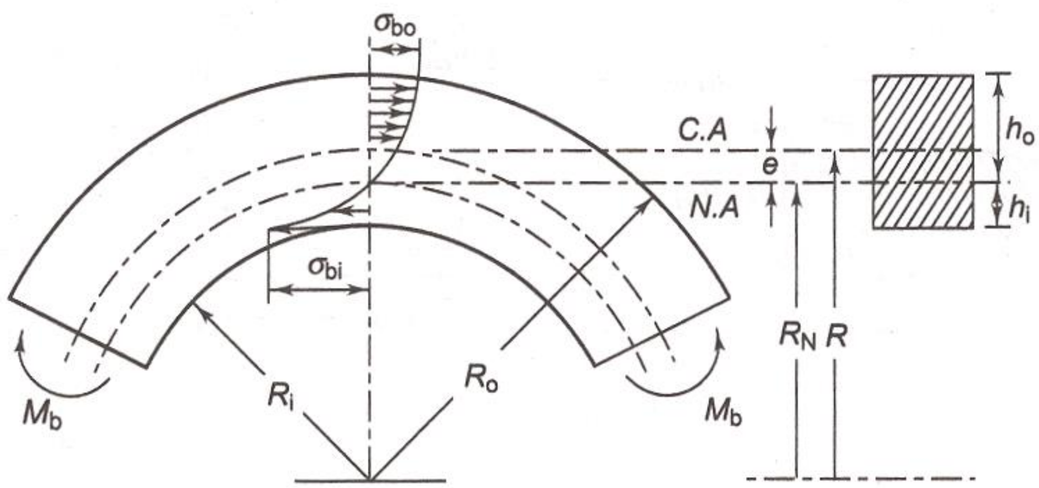


Fig. 7.2 . Stresses in Curved Beam (C.A. = ceritroidal axis; N.A. = neutral axis)

The eccentricity 'e' between centroidal and neutral axes is given by,

$$e = R - RN \qquad \ldots(1)$$

The bending stress (σ_b) at a fibre, which is at a distance of 'y' from the neutral axis is given by,

$$\sigma_b = \frac{M_b y}{Ae(R_N - y)} \ldots(2)$$

The equation indicates the hyperbolic distribution of (σ_b) with respect to ‘y’. The maximum stress occurs either at the inner fibre or at the outer fibre.

The bending stress at the inner fibre is given by,

$$\sigma_{b_i} = \frac{M_b h_i}{AeR_i} \ldots(3)$$

Similarly, **the bending stress at the outer fibre is given by,**

$$\sigma_{bo} = \frac{M_b h_o}{AeR_o} \ldots(4)$$

In symmetrical cross-sections, such as circular or rectangular, the maximum bending stress always occurs at the inner fibre. In unsymmetrical cross sections, it is necessary to calculate the stresses at the inner as well as outer fibres to determine the maximum stress. In most of the engineering problems, the magnitude of ‘e’ is very small and it should be calculated precisely to avoid large percentage error in the final results.

7.3. NOMENCLATURE OF CURVED BEAMS

The nomenclature for commonly used cross-sections of curved beams is illustrated in Fig. 7.3.

For rectangular cross-section [Fig. 7.3 (a)],

$$R_N = \frac{h}{\log_e\left(\frac{R_o}{R_i}\right)} \ldots(5)$$

and $R = R_i + \frac{h}{2}$ ...(6)

For circular cross-section [Fig. 7.3 (b)],

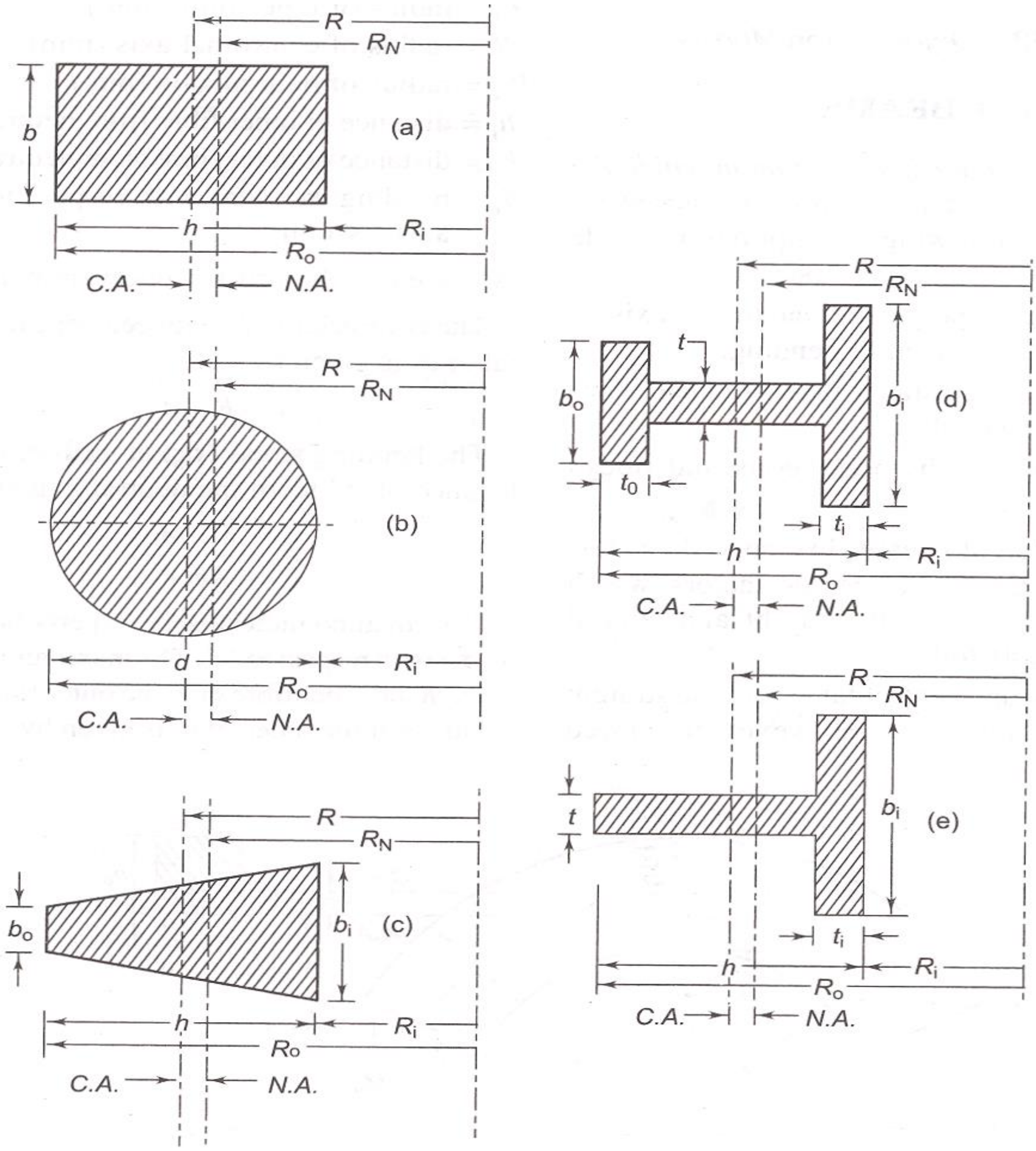
$$R_N = \frac{\left(\sqrt{R_o} + \sqrt{R_i}\right)^2}{4} \qquad \ldots(7)$$

and $R = R_i + \dfrac{d}{2}$ ...(8)

For trapezoidal cross-section [Fig. 7.3 (c)],

$$R_N = \frac{\left(\dfrac{b_i + b_o}{2}\right)h}{\left(\dfrac{b_i R_o - b_o R_i}{h}\right)\log_e\left(\dfrac{R_o}{R_i}\right) - (b_i - b_o)} \ldots(9)$$

and $R = R_i + \dfrac{h(b_i + 2b_o)}{3(b_i + b_o)}$ ...(10)



For I-section beam [Fig. 7.3 (d)],

$$R_N = \frac{t_i(b_i - t) + t_o(b_o - t) + th}{b_i \log_e\left(\frac{R_i + t_i}{R_i}\right) + t\log_e\left(\frac{R_o + t_o}{R_i + t_i}\right) + b_o \log_e\left(\frac{R_o}{R_o - t_o}\right)} \quad \ldots(11)$$

$$R = R_i + \frac{\frac{1}{2}th^2 + \frac{1}{2}t_i(b_i - t) + t_o(b_o - t)(h - t_o/2)}{t_i(b_i - t) + t_o(b_o - t) + th} \quad \ldots(12)$$

For T-section beam [Fig. 7.3 (e)],

$$R_N = \frac{t_i(b_i - t) + th}{(b_i - t)\log_e\left(\frac{R_i + t_i}{R_i}\right) + t\log_e\left(\frac{R_o}{R_i}\right)} \quad \ldots(13)$$

$$R = R_i + \frac{\frac{1}{2}th^2 + \frac{1}{2}t_i^2(b_i - t)}{th + t_i(b_i - t)} \quad \ldots(14)$$

<u>EXAMPLES</u>

1. A crane hook having an approximate trapezoidal cross-section is shown in Fig. 7.4. It is made of plain carbon steel 45C8 (S_{yt} = 380N/mm^2) and the factor of safety is 3.5. Determine the load carrying capacity of the hook.

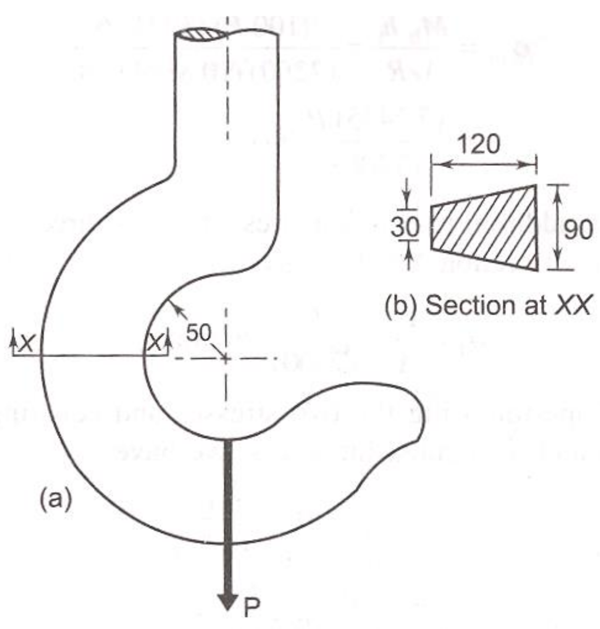


Fig.7.4.

Solution: For the cross-section XX [Eqs. (9) and (10)],

$$R_N = \frac{\left(\frac{b_i + b_o}{2}\right)h}{\left(\frac{b_i R_o - b_o R_i}{}\right)\log_e\left(\frac{R_o}{R_i}\right) - (b_i - b_o)}$$

$$R_N = \frac{\left(\frac{90+30}{2}\right)(120)}{\left(\frac{90 \times 170 - 30 \times 50}{120}\right)\log_e\left(\frac{170}{50}\right) - (90-30)}$$

$$= 89.1816\text{ mm}$$

$$R = R_i + \frac{h(b_i + 2b_o)}{3(b_i + b_o)}$$

$$= 50 + \frac{120(90 + 2 \times 30)}{3(90 + 30)} = 100\text{ mm}$$

e =R-R_N= 100 - 89.1816 = 10.8184mm

h_i =R_N– R=89.1816 – 50=39.1816mm

$$A = \frac{1}{2}\left[h(b_i + b_o)\right]$$

$$= \frac{1}{2}[(120)(90 + 30)] = 7200\text{ mm}^2$$

M_b =PR=(I100P)N-mm

From Eq. (3), the bending stress at the inner fiber is given by.

$$\sigma_{bi} = \frac{M_b\, h_i}{AeR_i} = \frac{(100\,P)\,(39.1816)}{(7200)\,(10.8184)\,(50)}$$

$$= \frac{(7.2435)\,P}{(7200)}\text{ N/mm}^2 \qquad \text{(i)}$$

In addition to bending stress, there is direct tensile stress at section XX. It is given by,

$$\sigma_t = \frac{P}{A} = \frac{P}{(7200)} \text{ N/mm}^2 \qquad \text{(ii)}$$

Superimposing the two stresses and equating the resultant to permissible stress, we have

$$\sigma_{bi} + \sigma_t = \frac{S_{yt}}{(fs)}$$

$$\frac{(7.2435)P}{7200} + \frac{P}{7200} = \frac{380}{3.5}$$

$$P = 94\,827.95 \text{ N}$$

2. A curved link of the mechanism made from a round steel bar, is shown in Fig. 7.5. The material of the link is plain carbon steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 3.5. Determine the dimensions of the link.

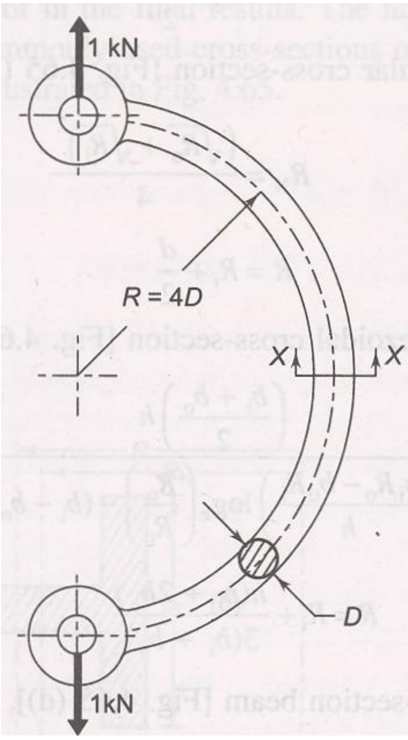


Fig.7.5

Solution

At section XX,

$R = 4D$

$R_1 = 4D - 0.5D = 3.5D$

$R_o = 4D + 0.5D = 4.5D$

From Eq. (7),

$$R_N = \frac{\left(\sqrt{R_o} + \sqrt{R_i}\right)^2}{4} = \frac{\left(\sqrt{4.5D} + \sqrt{3.5D}\right)^2}{4} = 3.9843\,D$$

$e = R - R_N = 4D - 3.9843D = 0.0157D$

$h_i = RN - R_i = 3.9843D - 3.5D = 0.4843D$

$A = \frac{\pi}{4} D^2 = (0.7854D^2)\ mm^2$

$M_b = 1000 \times 4D = (4000D)$ N-mm

From Eq. (3), the bending stress at the inner fiber is given by,

$$\sigma_{bi} = \frac{M_b h_i}{AeR_i} = \frac{(4000D)(0.4843D)}{(0.7854D^2)(0.0157D)(3.5D)}$$
$$= \left(\frac{44\ 886.51}{D^2}\right) \text{N/mm}^2$$

In addition to bending stress, there is direct tensile stress at section XX. It is given by,

$$\sigma_t = \frac{P}{A} = \frac{1000}{(0.7854\ D^2)} = \left(\frac{1273.24}{D^2}\right) \text{N/mm}^2 \qquad \text{(ii)}$$

Superimposing the bending and direct tensile stresses and equating the resultant stress to permissible stress, we have

$$\sigma_{bi} + \sigma_t = \frac{S_{yt}}{(fs)}$$
$$\left(\frac{44886.51}{D^2}\right) + \left(\frac{1273.24}{D^2}\right) = \frac{400}{3.5}$$
$$D = 20.10 \text{ mm}$$

3. The C-frame of a 100 kN capacity press is shown in Fig. 7.6. The material of the frame is grey cast iron FG 200 and the factor of safety is 3. Determine the dimensions of the frame.

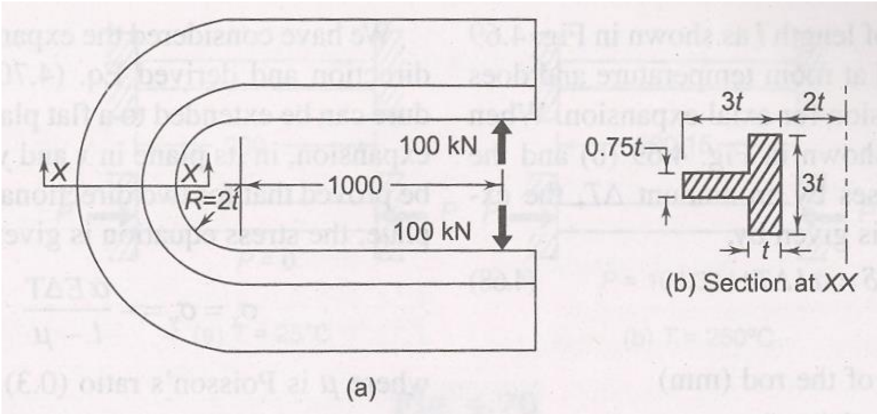


Fig. 7.6

Solution

The section at XX is subjected to direct tensile stress and bending stresses. Using notations of Eq. (13) and Fig. [7.3 (e)],

$b_i = 3t$ $\quad h = 3t$ $\quad R_i = 2t$

$R_o = 5t$ $\quad ti = t$ $\quad t = 0.75t$

From Eq. (13),

$$R_N = \frac{t_i(b_i - t) + th}{(b_i - t)\log_e\left(\frac{R_i + t_i}{R_i}\right) + t\log_e\left(\frac{R_o}{R_i}\right)}$$

$$= \frac{t(3t - 0.75t) + 0.75t(3t)}{(3t - 0.75t)\log_e\left(\frac{2t + t}{2t}\right) + 0.75t\log_e\left(\frac{5t}{2t}\right)}$$

$$= 2.8134t$$

From Eq. (14),

$$R = R_i + \frac{\frac{1}{2} th^2 + \frac{1}{2} t_i^2 (b_i - t)}{th + t_i(b_i - t)}$$

$$= 2t + \frac{\frac{1}{2}(0.75\,t)(3\,t)^2 + \frac{1}{2} t^2 (3\,t - 0.75\,t)}{(0.75\,t)(3\,t) + t\,(3\,t - 0.75\,t)} = 3t$$

e =R - R_N=3t - 2.8134t =0.1866t

h_i = R_N - R_i = 2.8134t - 2t =0.8134t

A = (3t)(t) + (0.75t)(2t) = (4.5t^2) mm^2

M_b = l00 x 10^3(1000+R)

= 100 x 10^3(1000 + 3t) N-mm

From Eq. (3), the bending stress at the inner fiber is given by,

$$\sigma_{bi} = \frac{M_b h_i}{A e R_i} = \frac{100 \times 10^3\ (1000 + 3\,t)(0.8134\,t)}{(4.5\,t^2)(0.1866\,t)(2\,t)}$$

$$= \frac{100 \times 10^3\ (1000 + 3\,t)(2.1795)}{(4.5\,t^2)}\ \text{N/mm}^2$$

The direct tensile stress is given by,

$$\sigma_t = \frac{P}{A} = \frac{100 \times 10^3}{(4.5\,t^2)}\ \text{N/mm}^2$$

Adding the two stresses and equating the resultant stress to permissible stress,

$$\sigma_{bi} + \sigma_t = \frac{S_{ut}}{(fs)}\ \frac{100 \times 10^3\ (1000 + 3\,t)(2.1795)}{(4.5\,t^2)}$$

$$+ \frac{100 \times 10^3}{(4.5t^2)} = \frac{200}{3}$$

$$t^3 - 2512.83\,t - 726\,500 = 0$$

Solving the above cubic equation by trial and errormethod,

$$t = 99.2\text{mm or } t = 100\text{mm}$$